Fast Identification of Loads and Damages Using a Limited Number Of Sensors

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ABSTRACT

This paper presents a method for fast identification of coexistent loads and damages, in which the number of sensors is mainly decided by the number of unknown loads. The computational efficiency is improved by the Virtual Distortion Method (VDM), which allows the repeated estimation of system impulse response to be performed efficiently, and by a local interpolation of perturbations of the structural response with respect to damage parameters. The proposed methodology is verified by a numerical example of a multi-span frame.

INTRODUCTION

External loads and structural damages are crucial factors in structural health monitoring, as well as in forensic engineering. Identification of either one of them has become an interesting topic with several effective methods. However, in a real application, unknown damages and unknown loads usually coexist and together influence the system response. Identification on either one of them can be interfered by the other one. Therefore, simultaneous identification of loads and damages seems to be a way to solve this problem.

It is in general not possible to identify the unknown load independently from the unknown damage. The existing load reconstruction methods demand a precalibrated model of the monitored structure whose structural parameters are known [1,2]. For damage identification methods [3], one can differentiate between model-based and pattern matching approaches. A part of them needs the external loads to be known. Others do not require exact information about the loads, but they are used in special conditions like ambient excitation or free response of the monitored structure. In case when both excitation and damage are unknown, the identification of either one is coupled to the identification of the other, so that they need to be identified

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together, which is usually performed in time domain with a large number of surplus sensors [4, 5].

This paper presents a new approach to simultaneous identification of unknown damages and excitations. In general, it only requires the number of sensors to be larger than that of the unknown external loads. Moreover, the optimization efficiency is improved by approximating the relation between the perturbations of the structural responses and the damage extents, and by constructing the relative system impulse response using the VDM [3].

SYSTEM IMPULSE RESPONSE VIA THE VDM

The Virtual Distortion Method is a quick reanalysis method. Structural damages are modeled by the related response-coupled virtual distortions or virtual forces. Then, the responses of the damaged structure can be quickly constructed by the linear combination of the intact structural responses to the same external load and to the virtual distortions, assuming that the intact system is known. Here, it is assumed that the damage affects both stiffness and mass of the involved structure.

Impulse Response of a Damaged Structure

Using the VDM, with the assumption of zero initial conditions, the impulse response of the damaged structure at the αth sensor to the ith external load can be expressed as the following:

$$h_{\alpha i}(t) = h_{\alpha i}^{L}(t) + \sum_{\tau \le t} \left(\sum_{\beta, j} D_{\alpha\beta j}^{\kappa}(t-\tau)\,\kappa_{\beta j}^{0}(\tau) + \sum_{\beta} D_{\alpha\beta}^{m}(t-\tau)\,p_{\beta}^{0}(\tau) \right) \tag{1}$$

where $h_{\alpha i}^{L}(t)$ is the corresponding impulse response of the intact structure, $D_{\alpha\beta j}^{\kappa}(t)$ or $D_{\alpha\beta}^{m}(t)$ are the impulse responses (*dynamic influence matrices* in the VDM terminology) of the αth sensor in the intact structure with respect to the jth distortion of the βth damaged element $\kappa_{\beta j}^{0}(t)$ and to the virtual force $p_{\beta}^{0}(t)$ applied in the βth degree of freedom (DOF) related to mass modifications.

In Eq.(1), $h_{\alpha i}^{L}(t)$ and the *dynamic influence matrices* can be computed in advance. The virtual distortions $\kappa_{\beta j}^{0}(t)$ and virtual forces $p_{\beta}^{0}(t)$, which model the damage, can be obtained via the expressions of the related distortion and acceleration responses of the damaged structure, which is similar to Eq.(1):

$$\kappa_{ij}(t) = \kappa_{ij}^{L}(t) + \sum_{\tau \le t} \left(\sum_{\beta, l} D_{ij\beta l}^{\kappa\kappa}(t-\tau)\kappa_{\beta l}^{0}(\tau) + \sum_{\beta} D_{ij\beta}^{\kappa m}(t-\tau)\,p_{\beta}^{0}(\tau) \right) \tag{2.a}$$

$$a_{i}(t) = a_{i}^{L}(t) + \sum_{\tau \le} \left(\sum_{\beta, l} D_{i\beta l}^{m\kappa}(t-\tau)\kappa_{\beta l}^{0}(\tau) + \sum_{\beta} D_{i\beta}^{mm}(t-\tau)\,p_{\beta}^{0}(\tau) \right) \tag{2.b}$$

where $\kappa_{ij}^{L}(t)$ and $a_{i}^{L}(t)$ are respectively the jth distortion coefficient of the ith damaged element and the acceleration of the ith DOF in the intact structure.

Similarly to $D^{\kappa}_{\alpha\beta j}(t)$ or $D^{m}_{\alpha\beta}(t)$, parameters $D^{\kappa\kappa}_{ij\beta l}(t)$ and $D^{m\kappa}_{i\beta l}(t)$ are the related *dynamic influence matrices*. In the VDM [3], one can prove that $\kappa^0_{ij} = (1-\mu_i)\kappa_{ij}$ and $p^0_i(t) = m_i a_i(t)$, therefore the unknowns $\kappa^0_{\beta j}(t)$ and $p^0_\beta(t)$ can be found by Eq.(2). The parameter μ_i is the stiffness-related damage extent of the *i*th element, that is the ratio of the modified stiffness to the original one.

In a discrete-time system, impulse responses to the *i*th external load in all the successive time steps of all the employed sensors can be expressed using Eq.(1) as:

$$\mathbf{h}_i = \mathbf{h}^{\mathrm{L}}_i + \mathbf{D}^{\kappa}\boldsymbol{\kappa}^0 + \mathbf{D}^{\mathrm{m}}\mathbf{p}^0 \tag{3}$$

where all the virtual distortions $\boldsymbol{\kappa}^0$ and forces $\mathbf{p}^0$ can be obtained from the discrete matrix form of Eq.(2), that is from

$$\left[\mathbf{I} - \begin{pmatrix} (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\kappa} & (\mathbf{I}-\boldsymbol{\mu})\mathbf{D}^{\kappa\mathrm{m}} \\ \mathbf{m}\mathbf{D}^{\mathrm{m}\kappa} & -\mathbf{m}\mathbf{D}^{\mathrm{mm}} \end{pmatrix}\right]\begin{bmatrix}\boldsymbol{\kappa}^0 \\ \mathbf{p}^0\end{bmatrix} = \begin{bmatrix}(\mathbf{I}-\boldsymbol{\mu})\boldsymbol{\kappa}^{\mathrm{L}} \\ \mathbf{m}\mathbf{a}^{\mathrm{L}}\end{bmatrix} \tag{4}$$

LOAD AND DAMAGE IDENTIFICATION

Via the constructed system impulse response, the measured structural response to the *i*th external load $p_i(t)$ can be expressed as:

$$y^{\mathrm{M}}_\alpha(t) = \sum_\tau \sum_i h_{\alpha i}(t-\tau)\, p_i(\tau) \tag{5}$$

In practice, the measured data and the computed impulse response are discrete. Hence, by collecting all the loads and all the measured responses (of all the sensors) in all considered time steps, Eq (5) can be rewritten in the matrix form as:

$$\mathbf{y}^M = \mathbf{H}\mathbf{p} \tag{6}$$

Generally, the unknown variables can be identified by minimizing the square distance between the estimated responses and the measured responses:

$$f(\mu, \mathbf{p}) = \left\| \mathbf{y}^{\mathbf{M}} - \mathbf{H}(\mu)\mathbf{p} \right\|^2 \tag{7}$$

In Eq (7), the system impulse responses $\mathbf{H}$ is the function of the parameters μ, which collectively define the damage, where $\mu = \left[\mu^{\bigstar}_1, \ldots, \mu^{\bigstar}_{n_m+n_e}\right]$ and $\mu^{\bigstar}_i = m_i / m^{tr}_i$ (if $i = 1, \ldots, n_m$) or $\mu^{\bigstar}_i = \mu_{i-n_m}$ (if $i=n_m+1, \ldots, n_m+n_e$). Here, n_m is the number of the mass modifications, n_e is the number of damaged element, m_i is the *i*th additional mass value and m^{tr}_i is its trial mass value, which may be estimated roughly via the measured response assuming the system is intact.

In other approaches [4,5], optimization of unknown damages and load time histories requires often a lot of optimization variables. However, given the damage extents, the structural impulse response can be constructed using the VDM. Then the corresponding external load $\mathbf{p}^e$ can be directly obtained via Eq.(6), possibly in a moving time window [4]. Therefore, only the damage extents can be treated as the optimization variables, and the number of sensors greater than the number of the loads is enough to ensure the uniqueness of the solution.

$$f(\mu) = \left\| \mathbf{y}^{\mathrm{M}} - \mathbf{H}(\mu)\mathbf{p}^e(\mu) \right\|^2 \tag{9}$$

Eq.(9) shows that the main task in each optimization step is to estimate the current system response. Hence, an approach of approximating the structural responses is used to improve the computational efficiency. The relation between the perturbation of the structural response $\Delta y(t)$ and of the damage extents $\Delta\mu_i$ is approximated using a certain function, like quadratic or splines, so that during the optimization the impulse-responses and the related reconstructed loads have to be calculated only few times to determine the approximation coefficients.

Here the relation is approximated by a quadratic curve. Based on given initial values of the damage extents, assume that the *i*th damage extent has two perturbations $\Delta\mu^{\star}_{i,j}, j = 1,2$. Denote by $\Delta y_{i,j}(t)$ the corresponding perturbation of the structural response relative to the initial response. Then the coefficients of the approximating functions between $\Delta y_{i,j}(t)$ and $\Delta\mu^{\star}_{i,j}$ can be got by:

$$\mathbf{A}_i(t) = \begin{bmatrix} 0 & \Delta y_{i,1}(t) & \Delta y_{i,2}(t) \end{bmatrix} \begin{bmatrix} 0 & \varsigma_{i,1} & \varsigma_{i,2} \end{bmatrix}^{-1} \tag{10}$$

where $\varsigma_{i,j} = \begin{bmatrix} \left(\Delta\mu^{\star}_{i,j}\right)^2 & \Delta\mu^{\star}_{i,j} & 1 \end{bmatrix}^T$ is the base vector of quadratic curve. Then, given any damage perturbation $\Delta\mu^{\star}_{i,m}$, the corresponding structural response perturbation $\Delta y_{i,m}(t)$ can be estimated by $\Delta y_{i,m}(t) = \mathbf{A}_i(t)\varsigma_{i,m}$.

In the case of small damages, it can be assumed that the perturbation of each damage extent contributes independently to the structural response. Thus, based on given initial damage extents, the unknown damage extents can be identified by computing the optimal value $\Delta\mu^{\star}_{i,m}$, which minimizes the objective function:

$$f = \left\| \sum_i \mathbf{A}_i \varsigma_{i,m} + \mathbf{y} - \mathbf{y}^{\mathrm{M}} \right\|^2 \tag{11}$$

where the coefficients in the *k*th row of the matrix $\mathbf{A}_i$ correspond to the approximating function at the *k*th time step, and $\mathbf{y}$ collects all discrete structural responses to the initial damage extents at all the sensors. In order to improve the accuracy, in each iteration the perturbations are halved.

NUMERICAL EXAMPLE

A numerical multi-span frame model in Figure 1 is used to verify the proposed method. Young's modulus is 2.15×10^{11}N/m^2, the density is 7800kg/m^3, and the beam inertia moment is 0.8m 4. The two poles are 20m high with an inertia moment of 0.16m^4. The beam is evenly divided into 20 elements, along with 2 even elements for each of the poles. An additional mass of 61.2×10^3kg is located on the beam 130m away from the left end. Elements No.21 and 23 are damaged with the stiffness reduction of μ_{21}=0.4, μ_{24}=0.7. Three sensors are employed: s1 at location 115.2m, s2 and s3 on the two columns 5.2m away from the neutral axis of the beam. A single excitation is applied on the beam 90m away from the left end (Figure 2). The sampling frequency is 100Hz, with the total sampling time 1s. Measurement errors of the simulated measured data are modeled by independent Gaussian noise at 5% rms level, shown in Figure 3.

Assume that the two poles are damaged with unknown stiffness reductions, which are to be identified besides the excitation and the additional mass. Hence, there are five damage extents to be identified and the unknown excitation. The initial trial value of the additional mass is 143×10^3kg. The damage extents are taken as optimization variables, with the unit initial values. The two perturbations of the stiffness-related damage extents are [-0.45, -0.9] in the first iteration, while the perturbations of the mass-related damage are [4, 6]. The iterations are performed 6 times altogether.

Table I lists the identified damage extents. The results are assessed respectively by the relative accuracy (mass) or the absolute accuracy (damage extents). Both the damage extents and their locations can be identified well even with 5% rms noise pollution. Figure 5 shows the evolution of the values in all iterations, and confirms that an accurate result could be obtained in only several iterations. Figure 6 shows the final reconstructed external load, which is very close to the actual load.

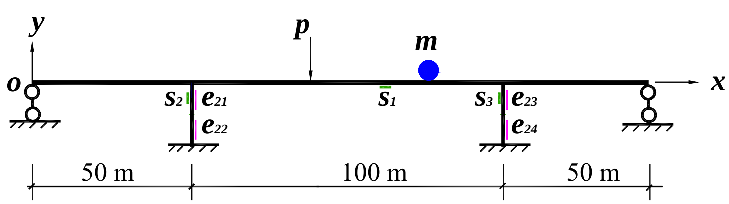

Figure 1. Damaged three-span frame structure

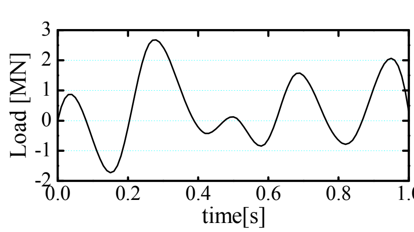

Figure 2. Assumed excitation

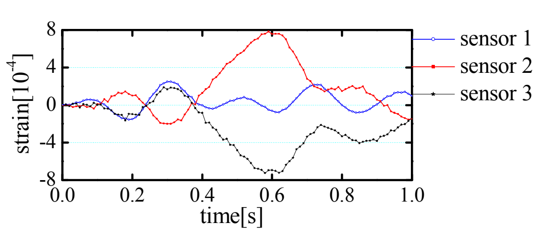

Figure 3. Noisy response of the damaged structure

TABLE I. IDENTIFIED DAMAGE EXTENTS

	μ_{21}	μ_{22}	μ_{23}	μ_{24}	m
identified	0.3865	0.9561	0.9759	0.6312	59.16×10^3kg
error(%)	1.35	4.39	2.41	6.88	3.33

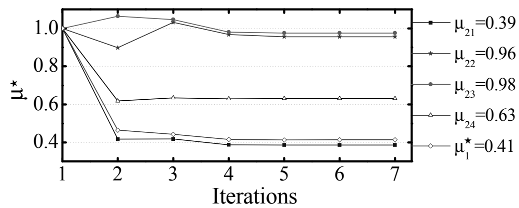

Figure 5. Optimization variables in successive iterations

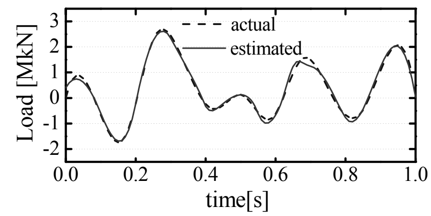

Figure 6. Actual and identified loads

CONCLUSIONS

An efficient method for identification of coexistent loads and damages has been proposed and validated in a numerical experiment of a mutli-span frame. Both stiffness-related and mass-related damages have been accurately identified together with the unknown load. Multiple damages and loads can be identified with fewer sensors.

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